Indian J. Phys. **54B**, 73–79 (1980)

Filamentation of very intense laser beams during their propagation in dielectrics

Satish C Abbi and Nitin C Kothari

Department of Physics, Indian Institute of Technology,
Hauz Khas, New Delhi-110029

Abstract. Some preliminary results regarding the estimation of filament formation thresholds in non-linear media are presented. Energy conservation plays a central role in estimating the distance D_f, for the first appearance of filaments. We present a theory in which the perturbations initially grow exponentially, separate from the main beam and their growth terminates. For a plano background and sinosoidal perturbations, this separation and growth termination is shown to be related to a local energy conservation criterion. For gaussian laser beams and gaussian perturbations, the termination of growth, relates to a more general energy conservation criteria. Our expressions for D_f for these two cases are different from those obtained by the Strong Field Theory of Suydam *et al*. Our theoretical results are in fair agreement with our experimental data and data from other authors.

1. Introduction

Theories (Chiao *et al* 1964, Akhmanov *et al* 1968, Marburger 1975, Svelto 1974) considering the propagation of a laser beam with smooth Gaussian intensity profile predict the existence of a very sharp (point) self-focal spot inside a medium whose dielectric constant has electric field intensity dependence of the type $\epsilon = \epsilon_0 + \epsilon_2 |E|^2$. The distance into the medium at which this occurs, i.e. the self-focal length Z_{sf} is given by the following expression :

$$1 \tag{1}$$

where $2r_0$ is the Gaussian laser beam width at half-maxima and $K = 2\pi/\lambda_0$ with λ_0 being the wavelength in vacuum. The theory concerns with the solution of the stationary quasi-optic equation

$$2iK \frac{\partial A}{\partial Z} = \nabla_\perp^2 A + \frac{\epsilon_2}{\epsilon_0} K^2 |A^2| A \tag{2}$$

under the paraxial ray approximation. Here A is the electric-field amplitude of a linearly polarized quasi-monochromatic wave propagating along z-direction, inside the medium, and $\nabla_\perp^2$ is a two dimensional Laplace Operator in the plane perpendicular to the beam axis z.

It was shown earlier (Abbi 1971) that for laser powers greater than several times the critical power (very high power laser beams), the experimental values Z_{sf} in various media neither show the electric field dependence of (1) nor agree with the absolute values predicted by this equation. It was also established that amplitude-phase-fluctuations (Abbi 1971, Abbi and Mahr 1971a, 1971b) present in the incident laser beam are responsible for filament formation. The linearized instability theories (Bespalov and Talanov 1966) considered the growth of amplitude perturbations in the non-linear medium. Considering the propagation of sinusoidal perturbation over a uniform background, Bespalov and Talanov showed that perturbations with size

$$K_{\perp} = \left[\frac{\epsilon_2}{\epsilon_0} K^2 |E_0|^2\right]^{\frac{1}{2}} \tag{3}$$

grow fastest and have the e-folding length

$$\Gamma = \frac{1}{h_0} = \frac{2\epsilon_0}{\epsilon_2} \frac{1}{K|E_0|^2} \tag{4}$$

where h_0 is the fastest exponential growth constant for optimum size perturbations The experimentally observed results indicate (Abbi and Mahr 1971a, 1971b) that the distance into the medium for the first appearance of filaments, D_f, is according to the expression

$$D_f = \frac{1}{h_0} \cdot \eta = \Gamma.\eta \tag{5}$$

where η is a factor that depends on the nature of perturbation and the characteristics of the main laser beam By a heuristic extrapolation of linearized instability theory Suydam (1973, 1974) has given an expression $\eta = \ln (p/\delta)$ where $\delta = \frac{|\delta E|}{|E|}$ is the phase perturbation in radians) and ρ has values between 1 and 5. According to him $\rho = 3$ gives best experimental fit. We show that for plane wave laser background and sinusodial perturbations, the linearized instability results can be logically extended to show that $\rho = 2$ for this case. For the laser beam and perturbation both having gaussian intensity profiles, analytic results for D_f are obtained.

2. Theory

2.1 *Review of propagation theory for smooth gaussian profile laser beams* (Svelto 1974)

In order to obtain solutions of the quasi-optic equation (2), complex optical field amplitude is expressed as (Svelto 1974)

$$A = A_0 e^{-iks} \tag{6}$$

where the real quantities A_0 and S (the eikonal of the wave) are both functions of $\mathbf{r}$. Equations (2) and (6) yield the following two equations for A_0 and S in cylindrical co-ordinate system :

$$2\frac{\partial S}{\partial Z}+\left(\frac{\partial S}{\partial r}\right)^2=\frac{\epsilon_2}{\epsilon_0}|A_0|^2+\frac{1}{K^2A_0}\nabla_\perp^2A_0$$

$$\frac{\partial A_0}{\partial Z}+\left(\frac{\partial S}{\partial r}\right)\left(\frac{\partial A_0}{\partial r}\right)+\frac{1}{2}A_0\nabla_\perp^2S=0. \tag{7}$$

Equations (7), which were worked out by Akhmanov *et al* (1968) constitute the fundamental equations of the 'whole beam self-focussing' treatment. We look for the aberrationless solutions of equation (7) under the paraxial ray approximation of the type

$$A_0^2(r,Z)=\frac{E_0^2}{f^2}\exp\left[-\frac{r^2}{r_0^2f^2}\right]$$

$$S(r,Z)=\phi(Z)+\frac{r^2}{2}\beta(Z). \tag{8}$$

For an incident beam with a plane wave front, the substitution of (8) in (7) leads to

$$f^2(Z)=1-\frac{Z^2}{Z^2_{sf}};\qquad \beta=\frac{1}{f}\frac{df}{dZ} \tag{9}$$

where Z_{sf} is given by (1). Z_{sf} is the equivalent focal length, which describes the effects due to self-focusing and diffraction together. At the distance $Z=Z_{sf}$ inside the medium the beam collapses to a point The theoretical estimate of the self-focusing length, Z_{sf}, needed for the filament formation is atleast one order of magnitude larger than the experimentally observed values.

2.2 *Review of Linearized Instability theory* (*LIT*) (Bespalov and Talanov 1966)

This theory considers the growth of sinusoidal amplitude or phase perturbations riding on a plane background laser beams. The net optical electric field amplitude is taken to have the form :

$$A=(E_0+e_1+ie_2)e^{-ikS} \tag{10}$$

where S from equation (7) is given by

$$S=\frac{1}{2}\frac{\epsilon_2}{\epsilon_0}|E_0|^2Z. \tag{11}$$

Under the conditions $|e|\ll|E_0|$.

Equations (2) and (10) leads to the following two coupled linearized equations for e_1 and e_2 :

$$\nabla_\perp^2 e_1 + 2K\frac{\partial e_2}{\partial Z} + \frac{2\epsilon_2}{\epsilon_0} K^2 |E_0|^2 e_1 = 0$$

$$\nabla_\perp^2 e_2 - 2K \frac{\partial e_1}{\partial Z} = 0 \tag{12}$$

We consider the sinusoidal perturbations of dimension $K_\perp$, (spatial freq $= K_\perp/\pi$) riding on a uniform background. For perturbations of type

$$e_{1,2} = e_{10,\,20} e^{i\boldsymbol{k}_\perp \cdot \boldsymbol{r}} e^{ik_\parallel z} \tag{13}$$

the solutions of the coupled equation (12) yield the form for $K_\parallel$ as

$$K_\parallel = -\frac{K_\perp^2}{4K^2}\left(\frac{2\epsilon_2}{\epsilon_0} K^2 |E_0|^2 - K_\perp^2\right). \tag{14}$$

Perturbations with transverse wave numbers $0 < K_\perp^2 < 2\epsilon_2/\epsilon_0 K^2 |E_0|^2$ are unstable with respect to Z, $(K_\parallel^2 = -h^2)$, where as perturbations of smaller scale $K_\perp^2 > 2\epsilon_2/\epsilon_0 K^2 |E_0|^2$ are stable; $(h^2 > 0)$. The growth constant for unstable perturbations is given by

$$h^2 = \frac{K_\perp^2}{4K^2}\left(\frac{2\epsilon_0}{c_0} K^2 |E_0|^2 - K_\perp^2\right) \tag{15}$$

and has the maximum value given by

$$h_0^2 = \frac{1}{2}\frac{\epsilon_2}{\epsilon_0} K |E_0|^2 \tag{16}$$

when $K_\perp$ is given by

$$K_\perp^2 = \frac{\epsilon_2}{\epsilon_0} K^2 |E_0|^2 \tag{17}$$

corresponding to characteristic scales $\pi/K_\perp$ and $1/h_0$. Bespalov and Talanov suggested that for sufficiently intense incident beam, one can choose

$$D_f = \frac{1}{h_0} = \frac{2\epsilon_0}{\epsilon_2} \cdot \frac{1}{K|E_0|^2} \tag{18}$$

to be the characteristic scale of instability development (occurrence of glowing filaments).

2.3 *Prediction of Thresholds for filament formation*

(a) *Plane Background with sinusoidal perturbations*

According to the Linearized instability theory the sinusoidal perturbations grow exponentially. In actual physical situations they must derive energy from the main laser beam. Since both the background and the sinusoidal perturbation are of infinite extend it is awkward to apply overall energy conservation criterion to this case. A local energy conservation criterion is more appropriate and this requires energy conservation over a characteristic area for the sinusoidal perturbation. This would lead to the equation :

$$E_0^2 \cdot \frac{\pi^2}{K_{\perp x}K_{\perp y}} = (|e_{10}^2+e_{20}^2|)e^{2hz}\left[\int_0^{\pi/k_{\perp x}} \sin^2 K_{\perp x} d.dx\right]\left[\int_0^{\pi/k_{\perp y}} \sin^2 K_{\perp y} y\, dy\right]$$

which, for $Z = D_f$, leads to

$$D_f = \frac{1}{h_0} \ln \left|\frac{2}{\delta}\right| \tag{19}$$

where

$$\delta = \frac{|\delta E_0|}{|E_0|} = \frac{|e^2_{10}+e^2_{20}|^{\frac{1}{2}}}{|E_0|}$$

Here we have $K^2_{\perp x}+K^2_{\perp y} = K_\perp^2$.

Equation (19) gives the expression for the distance into the medium for the first appearance of filaments

(b) *Gaussian Laser beams with quassian perturbations*

We next consider a composite laser beam consisting of two Gaussians. The major part of the optical energy is initially concentrated in the larger Gaussian beam and the smaller axially located Gaussian beam is treated as a perturbation To make the mathematics simpler we have taken the fluctuations in the intensity of the beam as a perturbation in the form of a single spike of Gaussian profile located at the centre.

We seek the solution of equation (2) when the input composite beam has the form

$$A = (A_0+e_1+ie_2)e^{-ikz} \tag{20}$$

The perturbation, $e = e_1+ie_2$ considered (Sodha *et al* 1976) is of (i) small scale length (as compared to the dimension of the beam), (ii) has small characteristic length for its growth (as compared to the focusing length of the main beam as a whole). These restrictions are necessary, because the intensity distribution of a Gaussian E.M. beam in a nonlinear cubic medium is not treated beyond the paraxial region.

The solutions for the quasi-optic equation (2) with the perturbation of the form

$$e_{1,2} = e_{10,20}\, e^{\alpha(z)} \exp\left[-\frac{r^2}{2b^2(Z)}\right] \tag{21}$$

have been obtained and would be published elsewhere (Abbi and Kothari). The results indicate that perturbations with size given by

$$\frac{1}{b^2(Z)} = \frac{1}{2}\frac{\epsilon_2}{\epsilon_0}K^2\frac{E_0^2}{f^2} + \frac{1}{r_0^2 f^2} \tag{22}$$

grow fastest. Their growth parameter $\alpha(Z)$ is gifen by

$$\alpha_0(Z) = \frac{K}{4}\frac{\epsilon_2}{\epsilon_0}|E_0|^2 Z_{sf} \ln\left(\frac{Z_{sf}+Z}{Z_{sf}-Z}\right) - \ln f(z) \tag{23}$$

where Z_{sf} is given by (1) and $f(z)$ by equation (8). It can be verified that

$$\lim_{z\to 0}\frac{\alpha_0(Z)}{Z} = h_0 = \frac{1}{2}\frac{\epsilon_2}{\epsilon_0}K|E_0|^2,$$

which is identical to the result of Linearized instability theory (Bespalov and Talanov 1966).

Overall Energy Conservation

In this case since both the main laser beam and the perturbation are of finite size one can apply the overall energy conservation criterion. The termination of growth would take place when the main laser beam energy is depleted and transferred to the energy of the perturbations. Setting up this criterion, leads to the equation.

$$\int_0^\infty E_0^2 f^{-2} e^{-r^2/r_0^2 f^2} \cdot r dr = \int_0^\infty (e_{10}^2 + e_{20}^2) e^{2\alpha_0(z)} e^{-r^2/b^2(z)}\, r dr \tag{24}$$

which gives

$$E_0^2 r_0^2 = \frac{(e_{10}^2+e_{20}^2)e^{2\alpha_0(z)} \cdot f^2}{\left(\frac{1}{2}\frac{\epsilon_2}{\epsilon_0}K^2|E_0|^2 r_0^2 + 1\right)}$$

or, for $Z = D_f$ we have, using equation (23)

$$D_f = \left\{ \frac{\left[\frac{1}{\delta}\left(\frac{1}{2}\frac{\epsilon_2}{\epsilon_0}K^2E_0^2r_0^2+1\right)^{\frac{1}{2}}\right]^{1/N}-1}{\left[\frac{1}{\delta}\left(\frac{1}{2}\frac{\epsilon_2}{\epsilon_0}K^2E_0^2r_0^2+1\right)^{\frac{1}{2}}\right]^{1/N}+1} \right\} Z_{sf} \tag{25}$$

where

$$N = \frac{K}{4}\frac{\epsilon_2}{\epsilon_0}|E_0|^2 Z_{sf} = \tfrac{1}{2} Z_{sf}.h_0.$$

Equation (25) relates the distance, D_f, for the first appearance of filaments to the self-focal length Z_{sf} for the smooth gaussian laser beam.

3. Comparison with Experimental Results and Discussion

The experimental values for D_f in nitrobenzene (Abbi and Mahr 1971a, 1971b) agree with the theoretical values from equation $D_f = 1/h_0.\eta$ with $\eta = 2{\cdot}3$. These results were presented earlier (Abbi and Mahr 1971a, 1972b) and are not repeated here. Equation (19) gives the value $\eta = \ln(2/\delta)$. Experimentally δ was found (Abbi and Mahr 1971a, 1971b) to have a value 0·1 ($\approx 1\%$ intensity in laser spikes) so that $\eta = 3$. Equation (25) gives a better fit to the experimental results

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